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HEAT EXCHANGE IN THERMALLY INITIAL PORTION OF TUBE WITH VARIABLE WALL TEMPERATURE

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An asymptotic solution is obtained for the laminar heat-exchange problem with variable wall temperature.

The heat problem for the boundary layer on the surface whose temperature follows the law

$$T_w = T_0 + Ax^\gamma, \tag{1}$$

possesses a self-similar solution; it was studied in detail in [1] primarily by asymptotic methods.

The thermally initial portion of the tube where the liquid temperature varies from its value at the wall T_W to the temperature of the flow core, T_0, is equal to the incoming temperature; this takes place in the region $\delta \ll d$ (see Fig. 1), and can be analyzed in the same way as a boundary layer; the self-similar solution of the heat problem can also be obtained.

In the case of T_W = const [$\gamma = 0$ in (1)] this solution was obtained first by Leveque [2]. Attempts to generalize this solution to the variable case were made by Leveque himself [2] and also by others in [3, 4] although Nu as a function of γ was not available as is the case in a boundary-layer problem.

In the present article the Leveque solution is directly generalized to the case of the wall temperature following a power law.

The heat equation for the thermally initial portion is given by [5]

$$\rho c_p u \frac{\partial T}{\partial x} = \lambda \frac{\partial^2 T}{\partial y^2}. \tag{2}$$

In a thin heat-exchange layer the liquid velocity can be regarded as proportional to y:

$$u = \beta y. \tag{3}$$

In particular, for laminar flow in a circular tube one has [5]

$$\beta = 8\bar{u}/d. \tag{4}$$

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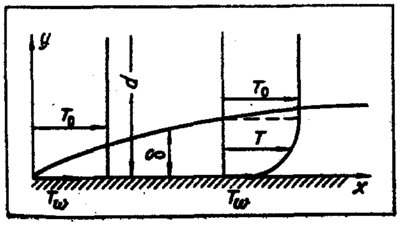

Fig. 1. Heat-exchange diagram in the thermally initial portion.

Equation (2) must be solved under the conditions

$$T = T_w \quad \text{for} \quad y = 0,$$
$$T \to T_0 \quad \text{for} \quad y \to \infty. \tag{5}$$

Adopting (1) for T_W we now introduce the dimensionless temperature

$$\theta = \frac{T - T_0}{T_w - T_0} \tag{6}$$

and the Leveque similarity variable [5]

$$\eta = (9\varkappa x)^{-1/3}\, y, \tag{7}$$

where

$$\varkappa = \lambda/(\beta\rho c_p). \tag{8}$$

Then (2) becomes an ordinary differential equation (primes denoting differentiation with respect to η)

$$\theta'' + 3\eta^2\theta' - 9\gamma\eta\theta = 0. \tag{9}$$

The conditions (5) now become

$$\theta = 1 \quad \text{for} \quad \eta = 0, \quad \theta \to 0 \quad \text{for} \quad \eta \to \infty. \tag{10}$$

Equation (9) is a particular case of the equation

$$\theta'' + a\eta^2\theta' + b\eta\theta = 0$$

for

$$a = 3, \quad b = -9\gamma, \tag{11}$$

which under the conditions (10) yields [6]

$$-\theta'(0) = \frac{\Gamma\left(\frac{2}{3}\right)\Gamma\left(1 - \frac{b}{3a}\right)}{\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{2}{3} - \frac{b}{3a}\right)} \left(\frac{a}{3}\right)^{\frac{1}{3}}. \tag{12}$$

Substituting (11) into (12) one obtains

$$-\theta'(0) = \frac{\Gamma\left(\frac{2}{3}\right)\Gamma(1+\gamma)}{\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{2}{3}+\gamma\right)}. \tag{13}$$

It follows from (1), (6), and (7) that

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{A x^{\gamma-\frac{1}{3}}}{(9\varkappa)^{1/3}}\,\theta'(0). \tag{14}$$

Introducing the Péclet number,

$$\mathrm{Pe} = \frac{\bar{u} d \rho c_p}{\lambda}, \tag{15}$$

and the local Nusselt number,

$$\mathrm{Nu} = \frac{-(\partial T/\partial y)_{y=0}\, d}{T_w - T_0} \tag{16}$$

and with the aid of (4), (8), and (13)-(15), one obtains

$$\mathrm{Nu} = \frac{2\Gamma\left(\frac{2}{3}\right)\Gamma(1+\gamma)}{9^{1/3}\,\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{2}{3}+\gamma\right)} \left(\mathrm{Pe}\,\frac{d}{x}\right)^{\frac{1}{3}}. \tag{17}$$

The formula (17) expresses the Nusselt number for a laminar flow in the thermally initial portion of a circular tube with the wall temperature following the law (1).

In the particular cases either of $\gamma = 0$ (wall temperature remains constant), or of $\gamma = 1/3$ (constant flow), or $\gamma = 1$ (wall temperature changing linearly) the results which follow from the formula (17) are the same as those obtained in [3, 4] by employing a different approach.

For $\gamma = 0$ the formula (17) yields the Leveque solution [5]

$$\mathrm{Nu} = 1.077\left(\mathrm{Pe}\,\frac{d}{x}\right)^{\frac{1}{3}}.$$

It is noted that in view of the assumption (3) and the form of Eq. (2) the solution (17) is an asymptotic one and becomes valid for $[\mathrm{Pe}(d/x)] \gg 1$ [5].

NOTATION

T, temperature of liquid; T_w, wall temperature; T_0, incoming temperature; A, γ, constants [formula (1)]; x, lengthwise coordinate; y, transverse coordinate; δ, thickness of thermal boundary layer; ρ, liquid density; c_p, specific heat; λ, thermal conductivity; u, velocity; $\bar{u}$, mean velocity; Pe, Péclet number; Nu, local Nusselt number.

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